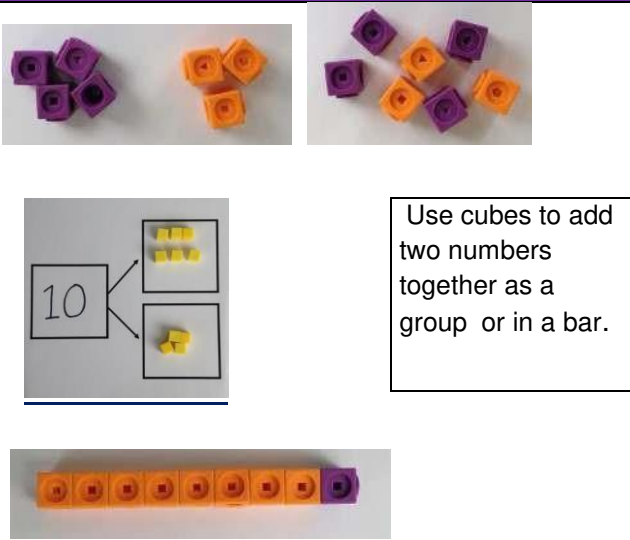
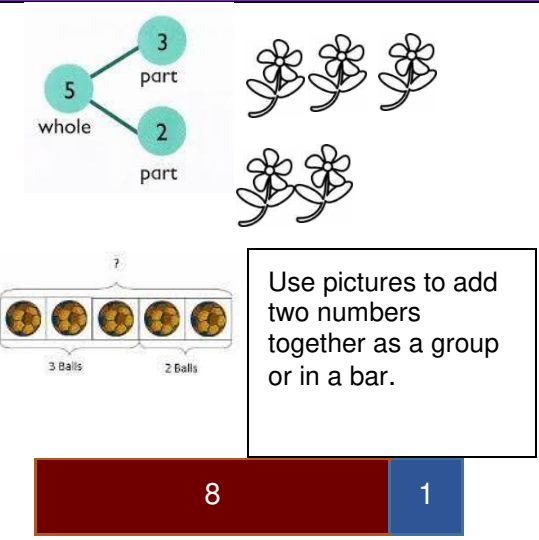
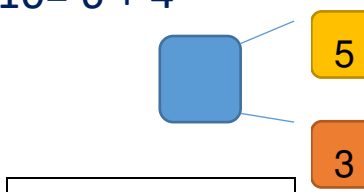

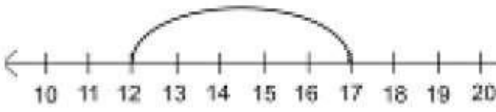




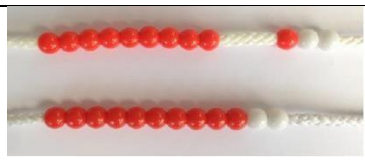
Fairfields Primary School

Progression in Calculation Policy

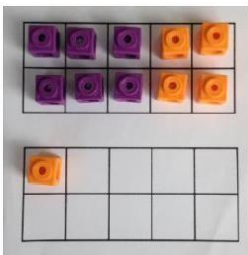
Progression in Calculations - Addition

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part-whole model</p>	 <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>$4 + 3 = 7$</p> <p>$10 = 6 + 4$</p>  <p>Use the part-part whole diagram as shown above to move into the abstract.</p>
<p>Starting at the bigger number and counting on</p>	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p>$12 + 5 = 17$</p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>$5 + 12 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p>

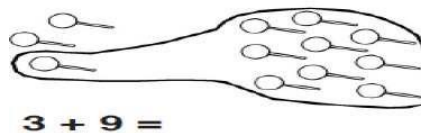
Regrouping to make 10



$$6 + 5 = 11$$

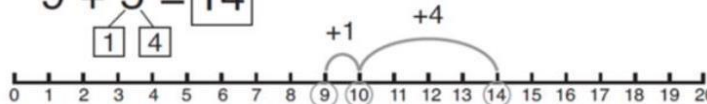


Start with the bigger number and use the smaller number to make 10.



Use pictures or a number line. Regroup or partition the smaller number to make 10.

$$9 + 5 = 14$$



$$7 + 4 = 11$$

If I am at seven, how many more do I need to make 10. How many more do I add on now?

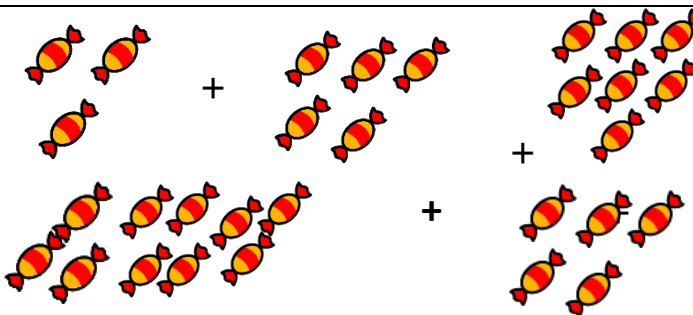
Adding three single digits

$$4 + 7 + 6 = 17$$

Put 4 and 6 together to make 10. Add on 7.



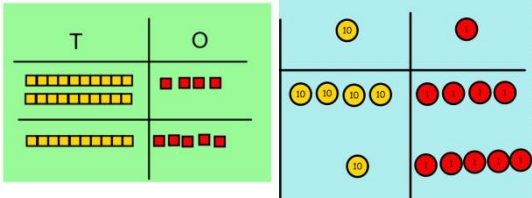
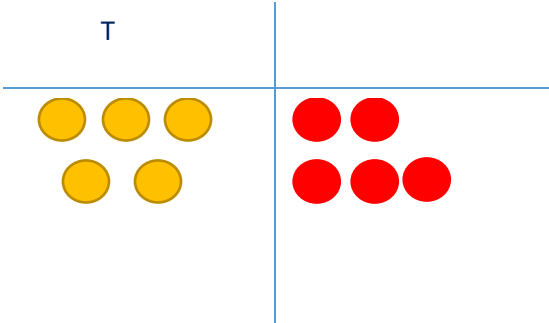
Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.



Add together three groups of objects. Draw a picture to recombine the groups to make 10.

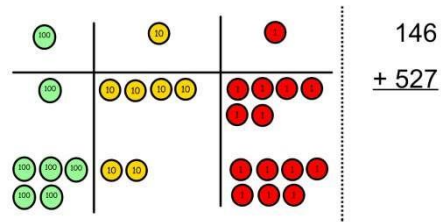
$$\begin{array}{r} 4 + 7 + 6 = 10 + 7 \\ 10 \\ = 17 \end{array}$$

Combine the two numbers that make 10 and then add on the remainder.

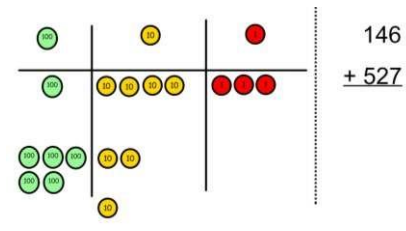
<p>Column method- no regrouping</p>	<p>$24 + 15 =$ Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.</p>  <p>The image shows two representations of the addition 24 + 15. On the left, a green grid represents base 10 blocks: two tens rods and four ones units in the 'T' column, and one ten rod and five ones units in the 'O' column. On the right, a blue grid represents place value counters: two tens circles and four ones dots in the 'T' column, and one ten circle and five ones dots in the 'O' column.</p>	<p>After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.</p>  <p>The image shows a place value chart with two columns labeled 'T' (Tens) and 'O' (Ones). In the 'T' column, there are two tens circles and four ones dots. In the 'O' column, there are one ten circle and five ones dots.</p>	<p><u>Calculations</u></p> <p>$21 + 42 =$</p> $\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$
--	--	--	--

Column method- regrouping

Make both numbers on a place value grid.



Add up the units and exchange 10 ones for one 10.

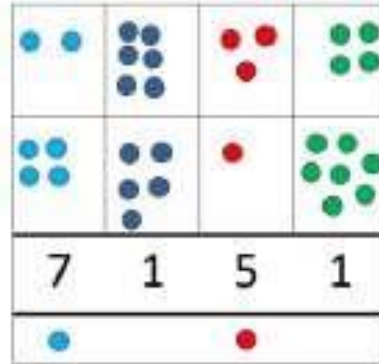


Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

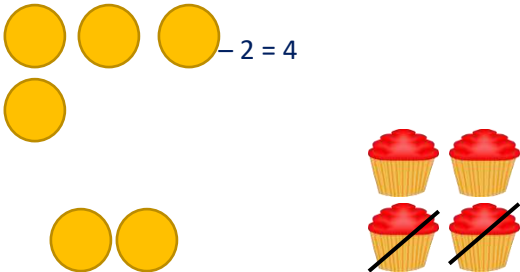
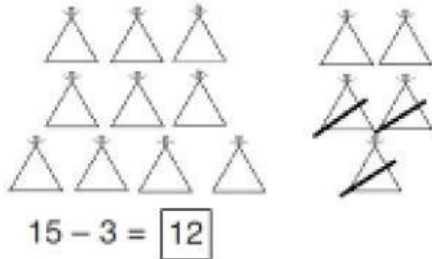


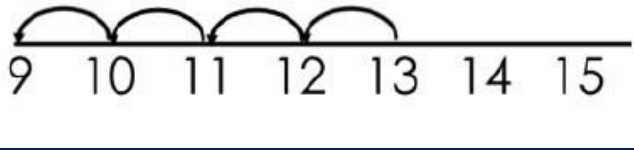
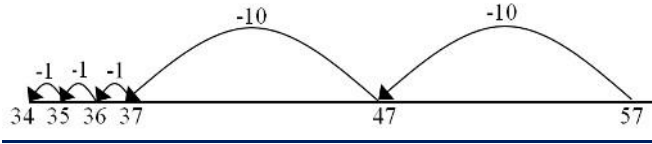
As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

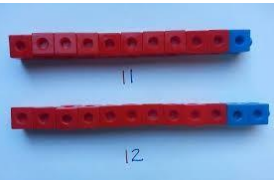
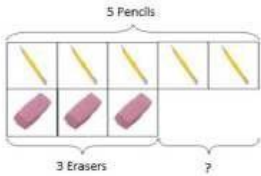
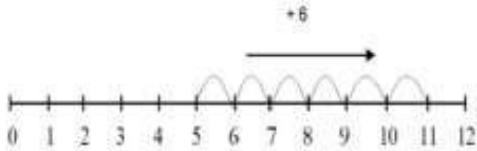
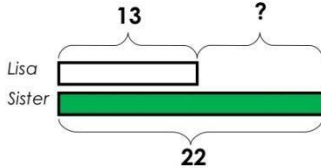
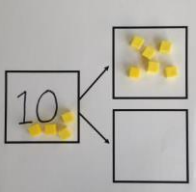
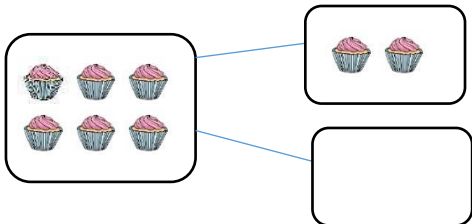
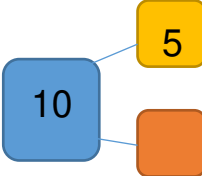

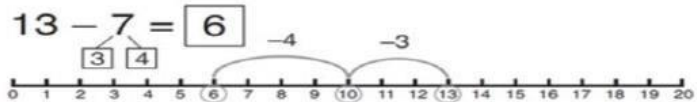
$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \end{array}$$

£	2	3	.	5	9
£	7	7	.	5	5
£	3	1	.	1	4
	1	1		1	

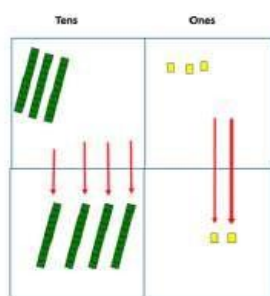
$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ 212 \end{array}$$

Progression in Calculations - Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p> 	<p>Cross out drawn objects to show what has been taken away.</p> 	$18 - 3 = 15$ $8 - 2 = 6$
Counting back	<p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p>  <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p> 	<p>Count back on a number line or number track</p>  <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p>  <p>This can progress all the way to counting back using two 2 digit numbers.</p>	<p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p>

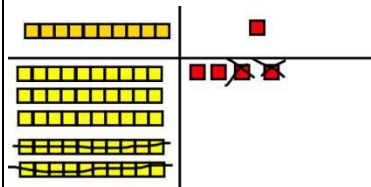
<p>Find the difference</p>	<p>Compare amounts and objects to find the difference.</p>  <p>Use cubes to build towers or make bars to find the difference</p>  <p>Use basic bar models with items to find the difference</p>	 <p>Count on to find the difference.</p> <p>Comparison Bar Models</p> <p>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</p>  <p>Draw bars to find the difference between 2 numbers.</p>	<p>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</p>
<p>Part -Part -Whole Model</p>	 <p>Link to addition- use the part whole model to help explain the inverse between addition and subtraction.</p> <p>If 10 is the whole and 6 is one of the parts. What is the other part?</p> <p>$10 - 6 =$</p>	<p>Use a pictorial representation of objects to show the part part whole model.</p> 	 <p>Move to using numbers within the part whole model.</p>
<p>Make 10</p>	 <p>$14 - 9 =$ Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.</p>	 <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p>	<p>$16 - 8 =$</p> <p>How many do we take off to reach the next 10?</p> <p>How many do we have left to take off?</p>

Column method without regrouping



Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.



Calculations

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

Draw the Base 10 or place value counters alongside the written calculation to help to show working.

$$\begin{array}{r} 47 - 24 = 23 \\ \begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array} \end{array}$$

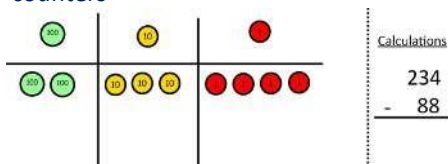
This will lead to a clear written column subtraction.

$$\begin{array}{r} 32 \\ - 12 \\ \hline 20 \end{array}$$

Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

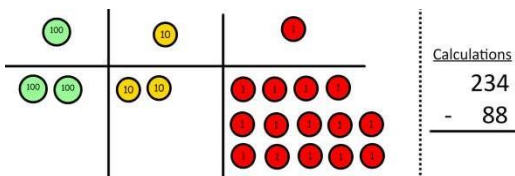
Make the larger number with the place value counters



Calculations

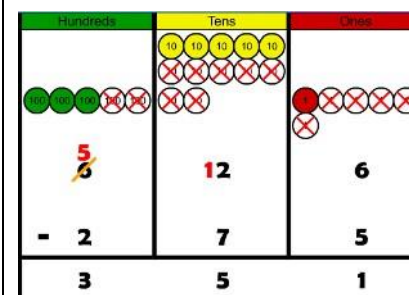
$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

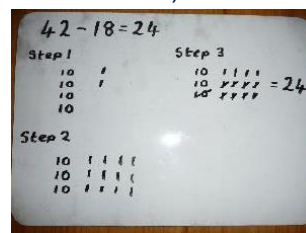


Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

$$\begin{array}{r} 836 - 254 = 582 \\ \begin{array}{r} \text{H} \quad \text{T} \quad \text{U} \\ 800 \quad 30 \quad 6 \\ - 200 \quad 50 \quad 4 \\ \hline 500 \quad 80 \quad 2 \end{array} \end{array}$$

Children can start their formal written method by partitioning the number into clear place value columns.

When confident, children can find their own way to record the exchange/regrouping.



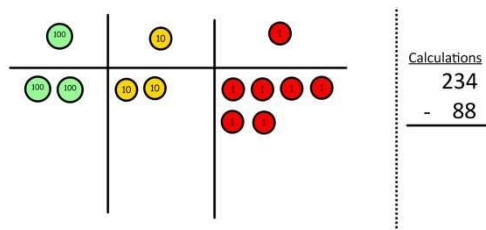
Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

$$\begin{array}{r} 728 - 582 = 146 \\ \begin{array}{r} \text{H} \quad \text{T} \quad \text{U} \\ 700 \quad 20 \quad 8 \\ - 500 \quad 80 \quad 2 \\ \hline 100 \quad 40 \quad 6 \end{array} \end{array}$$

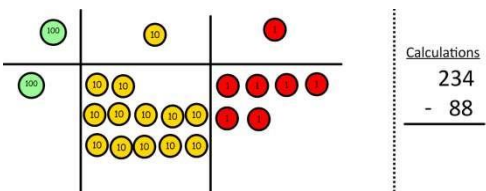
Moving forward the children use a more compact method.

--	--	--	--

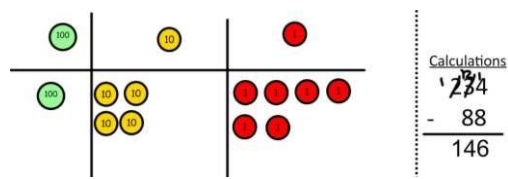
Now I can subtract my ones.



Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Now I can take away eight tens and complete my subtraction

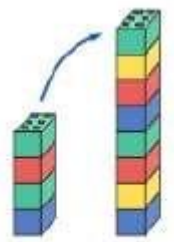

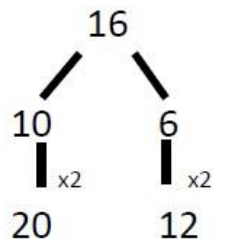


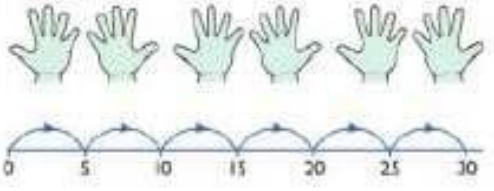


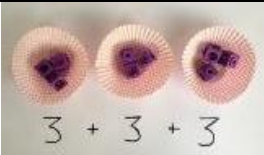



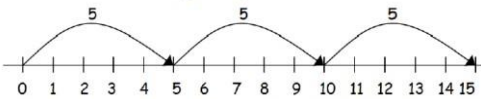



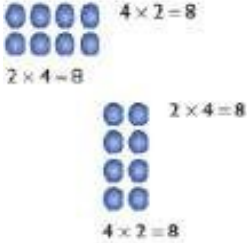
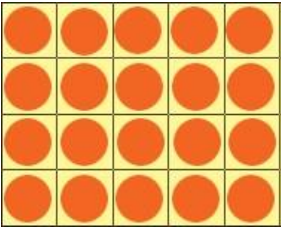

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

This will lead to an understanding of subtracting any number including decimals.

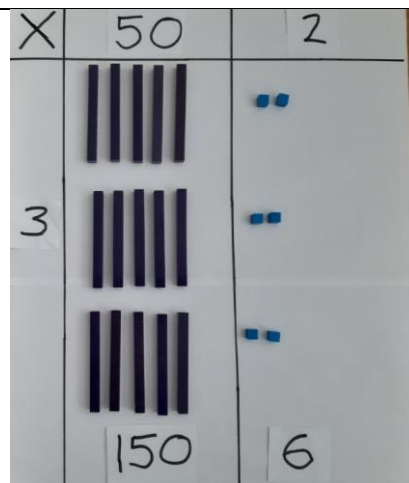
$$\begin{array}{r} 5 \quad 12 \quad 1 \\ 2 \quad \cancel{6} \quad \cancel{3} \quad . \quad \cancel{0} \\ - \quad 2 \quad 6 \quad . \quad 5 \\ \hline 2 \quad 3 \quad 6 \quad . \quad 5 \end{array}$$

Progression in Calculations - Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities to show how to double a number.</p>  <p>double 4 is 8 $4 \times 2 = 8$</p> <p>number.</p>	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together.</p>
Counting in multiples	  <p>Count in multiples supported by concrete objects in equal groups.</p>	 <p>Use a number line or pictures to continue support in counting in multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>

<p>Repeated addition</p>	<div data-bbox="338 65 600 220">  $3 + 3 + 3$ </div> <div data-bbox="622 86 851 268">  </div> <div data-bbox="349 272 600 389">  </div> <div data-bbox="636 323 828 430"> <p>Use different objects to add equal groups.</p> </div>	<p>There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?</p> <div data-bbox="891 181 1617 268">  </div> <p>2 add 2 add 2 equals 6</p> <hr/> <div data-bbox="909 379 1388 478">  </div> <p>$5 + 5 + 5 = 15$</p>	<p>Write addition sentences to describe objects and pictures.</p> <div data-bbox="1637 236 1989 331">  </div> <p>$2 + 2 + 2 + 2 + 2 = 10$</p>
<p>Arrays- showing commutative multiplication</p>	<p>Create arrays using counters/ cubes to show multiplication sentences.</p> <div data-bbox="331 563 741 874">  </div> <div data-bbox="580 887 864 1034">  </div>	<p>Draw arrays in different rotations to find commutative multiplication sentences.</p> <div data-bbox="1335 563 1581 807">  <p>$4 \times 2 = 8$ $2 \times 4 = 8$ $2 \times 4 = 8$ $4 \times 2 = 8$</p> </div> <div data-bbox="880 762 1160 989">  </div> <p>Link arrays to area of rectangles.</p>	<p>Use an array to write multiplication sentences and reinforce repeated addition.</p> <div data-bbox="1715 611 1939 762">  </div> <p>$5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$</p>

Grid method



Children will be taught a variety of methods to multiply. Grid method will be taught initially with concrete materials. Then move on to pictorially.

×	20	6
5	100	30

= 130

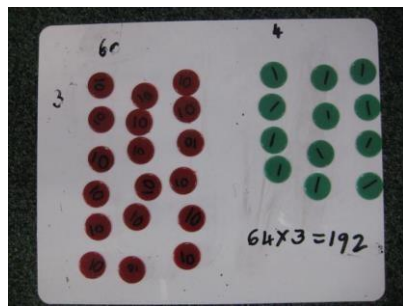
This is then moved into the abstract.

x	60	8
100	6000	800
50	3000	400
4	240	32

This can then be applied to 2 by 3 and 4 digit calculations.

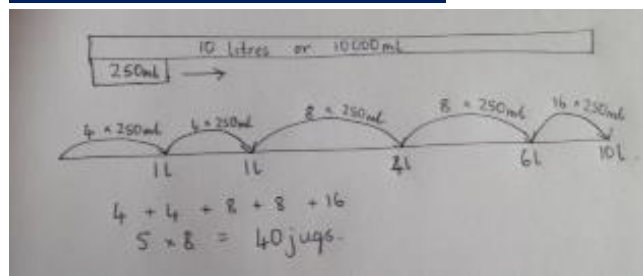
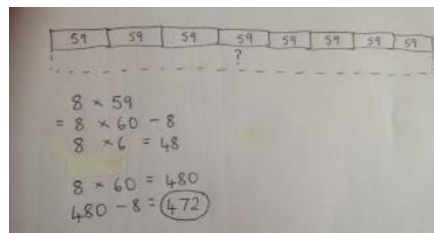
Column multiplication

Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

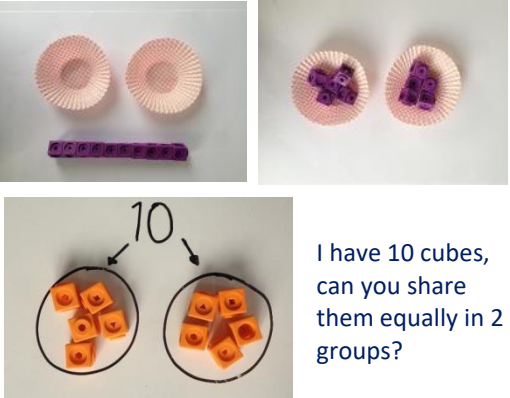
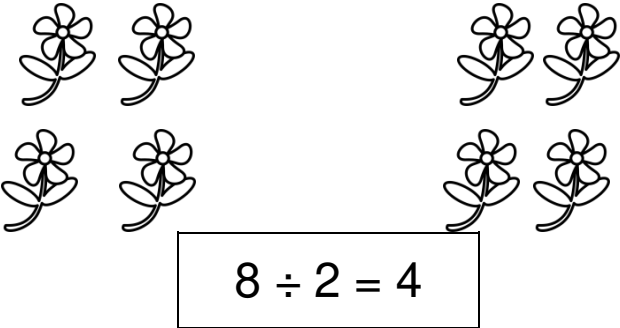
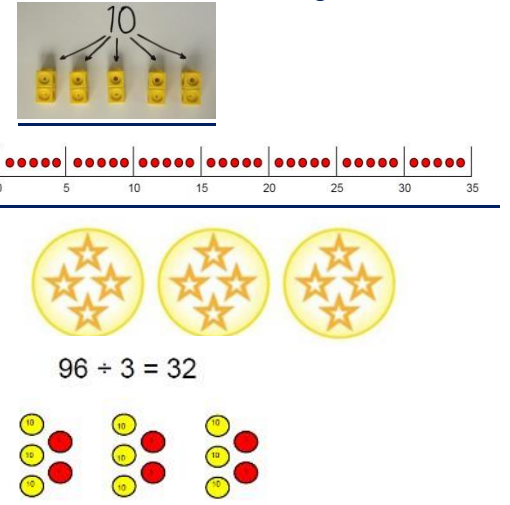
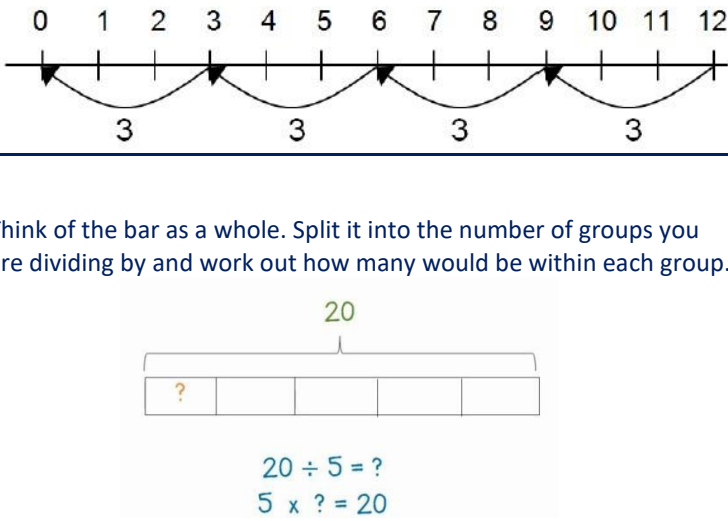
$$\begin{array}{r} 32 \\ \times 24 \\ \hline 8 \quad (4 \times 2) \\ 120 \quad (4 \times 30) \\ 40 \quad (20 \times 2) \\ 600 \quad (20 \times 30) \\ \hline 768 \end{array}$$

$$\begin{array}{r} 7 4 \\ \times 6 3 \\ \hline 1 2 \\ 2 1 0 \\ 2 4 0 \\ + 4 2 0 0 \\ \hline 4 6 6 2 \end{array}$$

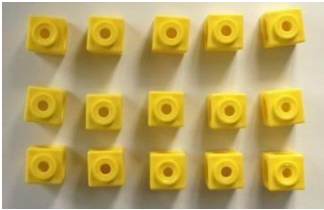
This moves to the more compact method.

$$\begin{array}{r} 2 3 1 \\ 1 3 4 2 \\ \times 1 8 \\ \hline 1 3 4 2 0 \\ 1 0 7 3 6 \\ \hline 2 4 1 5 6 \\ \hline 1 \end{array}$$

Progression in Calculations - Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  $8 \div 2 = 4$	<p>Share 9 buns between three people.</p> $9 \div 3 = 3$
Division as grouping	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>  $96 \div 3 = 32$	<p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p> $20 \div 5 = ?$ $5 \times ? = 20$	$28 \div 7 = 4$ <p>Divide 28 into 7 groups. How many are in each group?</p>

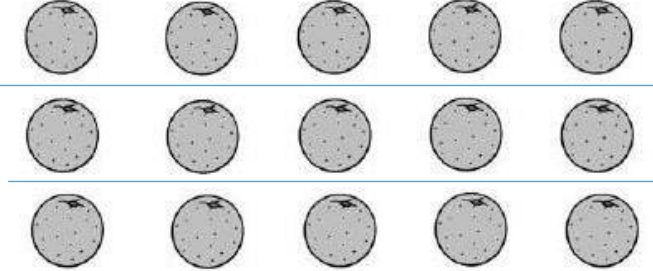
Division within arrays



Link division to multiplication by creating an array and thinking about the number sentences that

can be created.

Eg $15 \div 3 = 5$ $5 \times 3 = 15$
 $15 \div 5 = 3$ $3 \times 5 = 15$



Draw

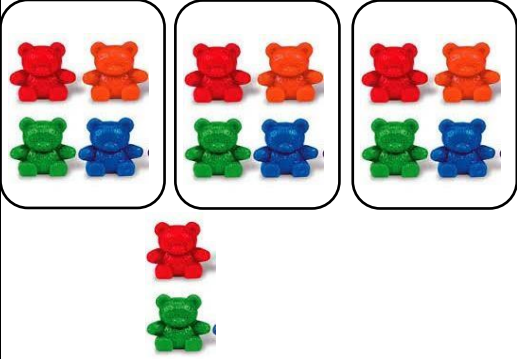
an array and use lines to split the array into groups to make multiplication and division sentences.

Find the inverse of multiplication and division sentences by creating four linking number sentences.

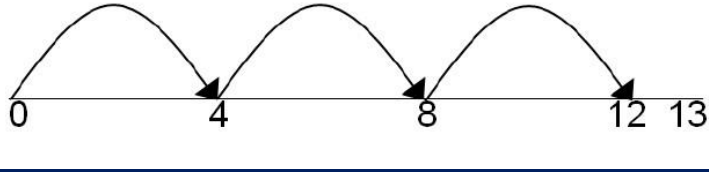
$7 \times 4 = 28$
 $4 \times 7 = 28$
 $28 \div 7 = 4$
 $28 \div 4 = 7$

Division with a remainder

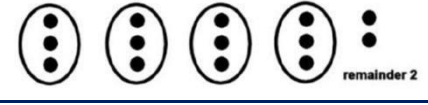
$14 \div 3 =$
 Divide objects between groups and see how much is left over



Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a



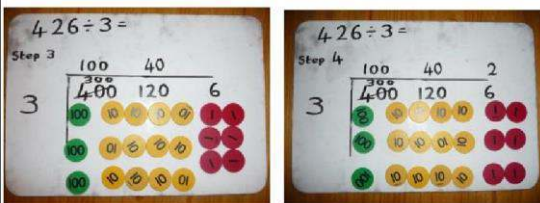
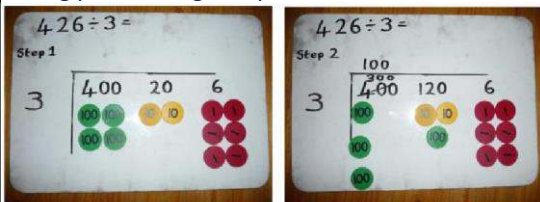
remainder.

Complete written divisions and show the remainder using r.

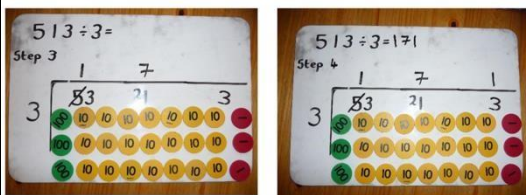
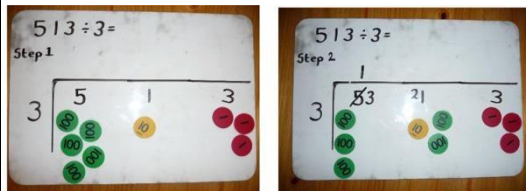
$29 \div 8 = 3 \text{ REMAINDER } 5$
 ↑ ↑ ↑ ↑
 dividend divisor quotient remainder

Short division

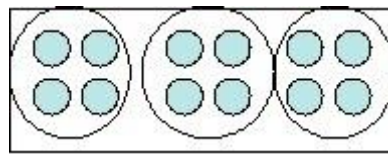
Divide a three digit number, then moving to a four digit number, by a one digit number by using partitioning and place value counters.



Divide a three digit number, then moving to a four digit number, by a one digit number without partitioning but using place value counters.



Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

Move onto divisions with a

$$\begin{array}{r} 218 \\ 3 \overline{) 862} \\ \underline{6} \\ 26 \\ \underline{24} \\ 20 \\ \underline{18} \\ 200 \\ \underline{180} \\ 200 \\ \underline{180} \\ 200 \\ \underline{180} \\ 200 \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \\ \underline{35} \\ 16 \\ \underline{15} \\ 11 \\ \underline{10} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

Long Division

Handwritten long division of 2974 by 13 on a piece of paper. The quotient is 228 with a remainder of 10. The work is shown in two columns. The left column shows the standard long division steps: 13 goes into 29 (2 times, remainder 13), 13 goes into 137 (10 times, remainder 7), 13 goes into 74 (5 times, remainder 9), and finally 13 goes into 91 (7 times, remainder 10). The right column shows the multiplication of the quotient digits by the divisor: 13 times 2 is 26, 13 times 10 is 130, 13 times 5 is 65, and 13 times 7 is 91. Arrows indicate the subtraction of these products from the dividend to find the next step.

$$\begin{array}{r} 0228,10 \\ 13 \overline{) 2974} \\ \underline{26} \\ 037 \\ \underline{26} \\ 114 \\ \underline{104} \\ 010 \end{array}$$
$$\begin{array}{r} 13 \\ +13 \\ \hline 26 \\ +13 \\ \hline 39 \\ +13 \\ \hline 52 \\ +13 \\ \hline 65 \\ +13 \\ \hline 78 \\ +13 \\ \hline 91 \\ +13 \\ \hline 104 \end{array}$$